Magnetostatics

Time variant magnetic feilds.

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

source of charge (scalar)

$$d\vec{D} = \frac{dq}{4\pi}\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

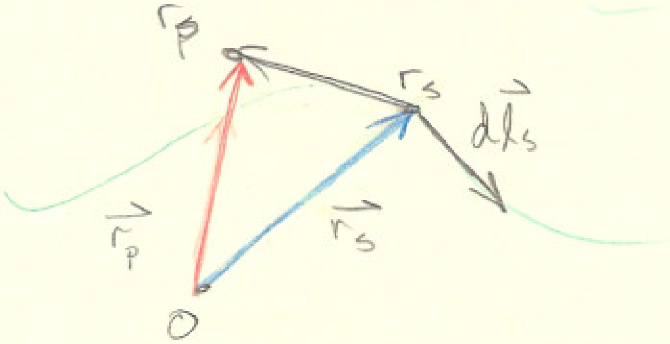
$$d\vec{H} = \frac{1}{4\pi}(I\,d\vec{l}_s)\times\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

cross product.

Small bit of current contained in "a"

The current contained in a small peice of wire of length $d\vec{l}$

$$\underbrace{(I\,d\vec{l})\times\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}}_{\text{results in a vector.}}$$



note: The resulting dH is into the page.

Length

$$d\vec{D} = \frac{1}{4\pi}(\lambda_s \, d\ell_s)\frac{(\vec{r}_p - \vec{r}_s)}{|\vec{r}_p - \vec{r}_s|^3}$$

$$d\vec{H} = \frac{1}{4\pi}(\underbrace{I}_{\text{Amps}} \, d\vec{\ell}_s) \times \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

notice the difference

Surface

$$d\vec{D} = \frac{1}{4\pi}(\sigma_s \, dA_s)\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

$$d\vec{H} = \frac{1}{4\pi}(\underbrace{\vec{K}}_{\text{Amp/meter}} \, dA_s) \times \frac{r_p - r_s}{|\vec{r}_p - \vec{r}_s|^3}$$

Volume.

$$d\vec{D} = \frac{1}{4\pi}(\rho_s \, dV_s)\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

$$d\vec{H} = \frac{1}{4\pi}(\underbrace{\vec{J}}_{\text{Amp/meter}^2} \, dV_s) \times \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|^3}$$

Not overly frequent in this course

EX: Infinite wire carrying constant current

current along $+\hat{z}$; Find $\vec{H}$ @ $\vec{r}_p = x_p\hat{x}$

For a small peice of wire @ $z_s\hat{z} = \vec{r}_s$

$$d\vec{\ell}_s = dz_s\hat{z}$$

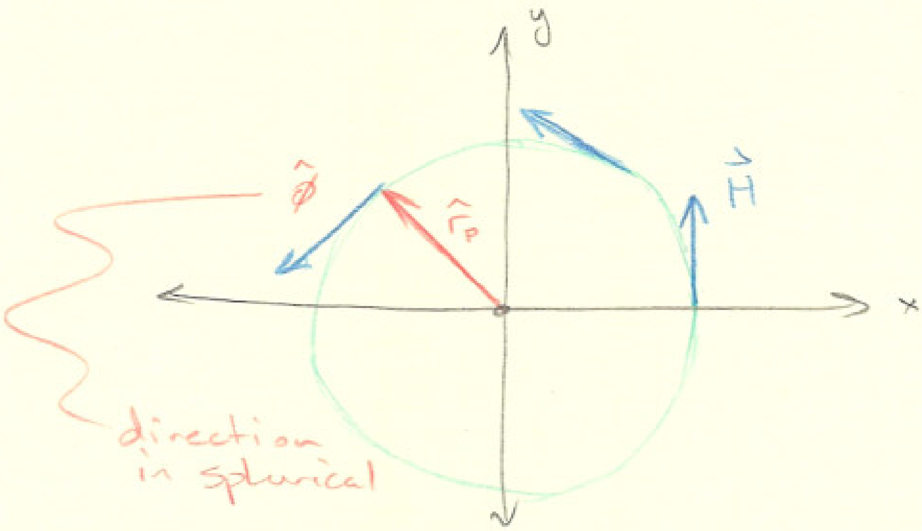
$$d\vec{H} = \frac{1}{4\pi}(I\,dz_s\,\hat{z}) \times \frac{x_p\hat{x} - z_s\hat{z}}{(x_p^2 + z_s^2)^{3/2}}$$

note: $\hat{z} \times \hat{z} = 0 \qquad \hat{z} \times \hat{x} = \hat{y}$

$$d\vec{H} = \frac{I\,dz_s}{4\pi}\frac{x_p}{(x_p^2 + z_s^2)^{3/2}}\hat{y}$$

$$\vec{H} = \hat{y}\int_{-\infty}^{\infty}\frac{I}{4\pi}x_p\frac{dz_s}{(x_p^2 + z_p^2)^{3/2}}$$

$$\vec{H} = \frac{I x_p \hat{y}}{4\pi}\int_{-\infty}^{\infty}\frac{dz_B}{(x_p^2 + z_p^2)^{3/2}} = \frac{I}{2\pi}\cdot\frac{1}{x_p}\hat{y}$$



$$\vec{H} = \frac{I}{2\pi}\frac{1}{r_p}\hat{\phi} \iff \vec{D} = \frac{q_s}{4\pi}\frac{1}{r_p}\hat{r}$$